



Modeling Reliability, Ambiguity, and Risk in Information Technology Management Using Quaternion Fuzzy and Quaternion Neutrosophic Graphs

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ARTICLE INFO

Article history:

Received 19 March 2026
Received in revised form 23 April 2026
Accepted 8 June 2026
Available online 11 December 2026

Keywords:

Information Technology Management; Reliability Analysis; Risk Management; Uncertainty Modeling; Quaternion Fuzzy Graphs; Quaternion Neutrosophic Graphs; Decision Support

ABSTRACT

Reliability assessment, ambiguity handling, and risk management are fundamental challenges in contemporary information technology management, where decision-making often relies on incomplete, uncertain, and heterogeneous information. Graph-based uncertainty models, including fuzzy and neutrosophic graphs, provide useful tools for representing dependencies and interactions among system components. However, complex IT environments frequently require richer representations capable of capturing multiple dimensions of uncertainty within a unified framework. To address this challenge, we introduce quaternion-based graph models for uncertainty representation in information technology management. Specifically, we develop *quaternion fuzzy graphs*, in which vertices and edges are labeled by elements of the quaternion unit ball and edge labels satisfy a norm-bounded incidence condition induced by their endpoints. We further propose *quaternion neutrosophic graphs*, where truth, indeterminacy, and falsity information are encoded through pure-quaternion components, providing a compact and expressive representation of uncertain relationships. We establish the fundamental properties of the proposed models and show that they preserve the corresponding quaternion fuzzy and quaternion neutrosophic set structures. Moreover, classical fuzzy graphs and single-valued neutrosophic graphs are recovered as special cases through canonical embeddings and componentwise reductions. The proposed framework offers a unified approach for modeling reliability, ambiguity, dependency strength, and risk in interconnected IT systems. Illustrative applications in IT Service Management and IT Project Management demonstrate its potential for representing complex dependencies and supporting uncertainty-aware analysis and decision-making in information technology management environments.

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1. Introduction

1.1 Fuzzy Graph and Neutrosophic Graph

Classical set theory treats membership as a two-valued relation: an element either belongs to a set or it does not. Although this binary viewpoint is mathematically fundamental, it is often too restrictive for situations in which available information is vague, partial, gradual, or mutually conflicting. Zadeh's fuzzy set theory provided one of the first systematic approaches to this limitation by replacing crisp membership with a membership degree in the unit interval [1]. Later, Atanassov's intuitionistic fuzzy sets refined this idea by considering membership and non-membership degrees simultaneously, thereby leaving room for hesitation or incomplete assessment [2, 3]. Neutrosophic set theory further broadened this line of development by assigning to each element three independent degrees, namely truth, indeterminacy, and falsity [4, 5]. This three-component description is especially useful when uncertainty is not merely imprecise, but also incomplete, inconsistent, or partially contradictory [6]. Moreover, further extended frameworks are also known, such as Quadripartitioned Neutrosophic Sets [7, 8], Bipolar Neutrosophic Sets [9, 10], Hesitant Neutrosophic Sets [11], Double-valued Neutrosophic Sets [12, 13], Uncertain Sets [14, 15], and Plithogenic Sets.

The development of these uncertainty-based frameworks has also had a significant impact on graph theory. In the setting of fuzzy graphs, membership degrees are assigned to both vertices and edges, making it possible to describe adjacency, incidence, and connectivity as matters of degree rather than as strictly present or absent relations [16]. Intuitionistic fuzzy graphs extend this framework by incorporating non-membership information together with membership information [17]. Neutrosophic graphs add a further level of expressiveness by explicitly representing indeterminacy in addition to truth and falsity components [18–20]. Related developments, such as plithogenic graphs, have also been introduced to describe systems in which several attributes and their possible contradiction effects must be considered simultaneously [21, 22]. These models show that graph-theoretic structures can be substantially enriched when uncertainty is treated as part of the vertex and edge data. For reference, a comparison of classical graphs, fuzzy graphs, and neutrosophic graphs is presented in Table 1.

1.2 Quaternion Neutrosophic Set

A different but complementary source of expressive power is provided by quaternion algebra. Quaternions extend complex numbers to a four-dimensional real algebra with one real component and three imaginary units [23, 24]. Their noncommutative multiplication and multidimensional structure make them suitable for describing information with directional, spatial, phase-like, or multi-component behavior. For this reason, quaternion-based models have appeared in several areas where scalar quantities are insufficient to represent the underlying structure of the data. In the context of uncertainty modeling, quaternion fuzzy sets allow fuzzy information to be considered in a quaternionic setting [25–27], while quaternion neutrosophic sets encode truth, indeterminacy, and falsity degrees through quaternionic components [28].

1.3 Our Contributions

Despite these developments, the graph-theoretic use of quaternion-based fuzzy and neutrosophic information has not been systematically formulated. Existing fuzzy and neutrosophic graph models generally rely on scalar membership values or scalar triples, whereas quaternionic labels can retain additional multi-component information that may be lost under scalar representation. This obser-

Table 1
Comparison of Classical Graphs, Fuzzy Graphs, and Neutrosophic Graphs.

Aspect	Classical Graph	Fuzzy Graph	Neutrosophic Graph
Basic structure	A graph with crisp vertices and edges.	A graph whose vertices and edges have membership degrees.	A graph whose vertices and edges have truth, indeterminacy, and falsity degrees.
Vertex representation	A vertex either belongs to the graph or does not.	Each vertex has a membership value in $[0, 1]$.	Each vertex is described by a triple $(T, I, F) \in [0, 1]^3$.
Edge representation	An edge is either present or absent.	Each edge has a membership value in $[0, 1]$.	Each edge is described by a triple $(T, I, F) \in [0, 1]^3$.
Uncertainty handling	No uncertainty is explicitly represented.	Represents graded or partial membership.	Represents truth, indeterminacy, and falsity simultaneously.
Main advantage	Simple and mathematically fundamental.	Suitable for systems with graded relationships.	Suitable for systems involving vague, incomplete, and inconsistent information.

vation motivates the present study. We introduce two graph models: *quaternion fuzzy graphs* and *quaternion neutrosophic graphs*. In a quaternion fuzzy graph, vertices and edges are labeled by elements of the quaternion unit ball, and the strength of each edge is controlled by the magnitudes of its incident vertices. In a quaternion neutrosophic graph, the truth, indeterminacy, and falsity degrees of vertices and edges are represented as pure-quaternion components.

The main contributions of this paper are as follows. First, we define quaternion fuzzy graphs in a norm-compatible manner and show that ordinary fuzzy graphs are recovered through the canonical embedding of scalar membership values into the real part of the quaternion algebra. Second, we introduce quaternion neutrosophic graphs by encoding neutrosophic triples as pure quaternions and prove that single-valued neutrosophic graphs arise as the corresponding componentwise reductions. Third, we provide concrete examples and illustrative interpretations in information technology management, including IT service management and IT project management, to demonstrate how the proposed models can describe reliability, ambiguity, dependency strength, and risk within a unified quaternionic graph framework.

2. Preliminaries

This section summarizes the notation and background concepts needed in the sequel. We first recall fuzzy graphs and single-valued neutrosophic graphs, which serve as the scalar and three-component graph-theoretic bases for the quaternionic models introduced later.

2.1 Fuzzy and Neutrosophic Graphs

Fuzzy graphs extend ordinary graph structures by assigning degrees of membership to vertices and edges. Thus, adjacency and incidence are no longer treated only as present or absent, but may be described by graded values reflecting uncertain or partial connectivity [29, 30]. In a related direction, neutrosophic graphs and neutrosophic hypergraphs enrich classical incidence structures by attaching

truth, indeterminacy, and falsity information to vertices, edges, or incidences; see, for example, [6, 31].

Definition 2.1 (Fuzzy graph). [16] Let V be a set of vertices. A *fuzzy graph* with vertex set V is an ordered pair

$$G = (\rho, \beta),$$

where

$$\rho : V \longrightarrow [0, 1]$$

assigns a membership degree to each vertex, and

$$\beta : V \times V \longrightarrow [0, 1]$$

assigns a membership degree to each possible edge. The edge-membership function β is assumed to be compatible with the vertex-membership function ρ . More precisely, for all $x, y \in V$, it satisfies

$$\beta(x, y) \leq \rho(x) \wedge \rho(y),$$

where $\wedge$ denotes the minimum of two real numbers. Thus, the degree assigned to an edge cannot exceed the smaller of the membership degrees of its two endpoint vertices.

The underlying crisp graph associated with G is written as

$$G^* = (\rho^*, \beta^*),$$

where

$$\rho^* = \{x \in V \mid \rho(x) > 0\}, \quad \beta^* = \{(x, y) \in V \times V \mid \beta(x, y) > 0\}.$$

Here, ρ^* consists of the vertices having positive membership degree, while β^* consists of the vertex pairs whose edge-membership degree is positive.

A *fuzzy subgraph* of G is a pair

$$H = (\rho', \beta')$$

defined over a subset $X \subseteq V$, where

$$\rho' : X \longrightarrow [0, 1]$$

is the vertex-membership function on X , and

$$\beta' : X \times X \longrightarrow [0, 1]$$

is the corresponding edge-membership function. These functions are required to satisfy

$$\beta'(x, y) \leq \rho'(x) \wedge \rho'(y), \quad \forall x, y \in X.$$

Hence, H again satisfies the same fuzzy incidence condition on the smaller vertex domain X .

Definition 2.2 (Single-valued neutrosophic graph (SVNG)). [6] Let

$$G^* = (V, E)$$

be a crisp graph, where V denotes the set of vertices and $E \subseteq V \times V$ denotes the set of edges. When G^* is directed, each element of E is understood as an ordered pair.

A *single-valued neutrosophic graph* on G^* is an ordered pair

$$G = (A, B),$$

where A assigns neutrosophic information to vertices and B assigns neutrosophic information to edges.

The vertex labeling is expressed as

$$A = \{ \langle v, TR_A(v), IND_A(v), FA_A(v) \rangle \mid v \in V \},$$

where

$$TR_A, IND_A, FA_A : V \longrightarrow [0, 1].$$

For each vertex $v \in V$, the quantities $TR_A(v)$, $IND_A(v)$, and $FA_A(v)$ represent, respectively, the truth-membership degree, the indeterminacy-membership degree, and the falsity-membership degree of v . They are required to satisfy

$$0 \leq TR_A(v) + IND_A(v) + FA_A(v) \leq 3, \quad \forall v \in V.$$

Similarly, the edge labeling is given by

$$B = \{ \langle uv, TR_B(uv), IND_B(uv), FA_B(uv) \rangle \mid uv \in E \},$$

where

$$TR_B, IND_B, FA_B : E \longrightarrow [0, 1].$$

For each edge $uv \in E$, the edge degrees must be compatible with the corresponding endpoint degrees. More precisely, the following conditions are imposed:

$$TR_B(uv) \leq \min\{TR_A(u), TR_A(v)\},$$

$$IND_B(uv) \leq \min\{IND_A(u), IND_A(v)\},$$

and

$$FA_B(uv) \geq \max\{FA_A(u), FA_A(v)\}.$$

Thus, the truth and indeterminacy degrees of an edge cannot exceed the smaller of the corresponding degrees of its endpoints, while the falsity degree of an edge must be at least as large as the greater falsity degree of its endpoints.

If the edge labeling satisfies the symmetry conditions

$$TR_B(uv) = TR_B(vu), \quad IND_B(uv) = IND_B(vu), \quad FA_B(uv) = FA_B(vu),$$

then G is called an *undirected* single-valued neutrosophic graph. If these equalities are not required or do not hold, then G is regarded as a *directed* single-valued neutrosophic graph.

2.2 Quaternion Neutrosophic Sets

A quaternion fuzzy set assigns each quaternion a membership grade in $[0,1]$, enabling fuzzy modeling of four dimensional values robustly. A quaternion neutrosophic set maps each element to truth, indeterminacy, and falsity degrees encoded as imaginary quaternion components in practice.

Definition 2.3 (Quaternion fuzzy set on $\mathbb{H}$). A *quaternion fuzzy set* (QFS) over the quaternion algebra $\mathbb{H}$ is a fuzzy membership assignment on $\mathbb{H}$. More precisely, it is a mapping

$$u : \mathbb{H} \longrightarrow [0, 1],$$

where $u(q)$ indicates the degree to which the quaternion $q \in \mathbb{H}$ belongs to the corresponding fuzzy quaternion object. Equivalently, this object may be written in the form

$$\tilde{Q} = \{ (q, u(q)) \mid q \in \mathbb{H} \}.$$

One standard way to construct such a set is to start from four real fuzzy numbers. Let

$$\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D} : \mathbb{R} \rightarrow [0, 1]$$

be fuzzy numbers on the real line. For a quaternion

$$q = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} \in \mathbb{H},$$

define

$$u(q) = u(a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k}) := \min\{\tilde{A}(a), \tilde{B}(b), \tilde{C}(c), \tilde{D}(d)\}.$$

Then

$$\tilde{Q} = \{ (q, u(q)) \mid q \in \mathbb{H} \}$$

forms a quaternion fuzzy set on $\mathbb{H}$.

If the membership function u is normal, upper semicontinuous, compactly supported, and convex in the level-set sense, then $\tilde{Q}$ is called a *fuzzy quaternion number*. In particular, the level-set convexity means that, for every $\alpha \in [0, 1]$, the α -cut

$$\{ q \in \mathbb{H} \mid u(q) \geq \alpha \}$$

is a convex subset of $\mathbb{R}^4$, after identifying $\mathbb{H}$ with $\mathbb{R}^4$.

Definition 2.4 (Fuzzy pure quaternion set). Let

$$\text{Im}(\mathbb{H}) := \{ b\mathbf{i} + c\mathbf{j} + d\mathbf{k} \mid b, c, d \in \mathbb{R} \}$$

denote the set of pure quaternions, that is, the quaternions whose real part is zero. A *fuzzy pure quaternion set* is a fuzzy membership function defined on $\text{Im}(\mathbb{H})$. More precisely, it is a mapping

$$u : \text{Im}(\mathbb{H}) \longrightarrow [0, 1].$$

A typical construction can be obtained from three real fuzzy numbers. Let

$$\tilde{B}, \tilde{C}, \tilde{D} : \mathbb{R} \rightarrow [0, 1]$$

be fuzzy numbers on the real line. For a pure quaternion

$$q = b\mathbf{i} + c\mathbf{j} + d\mathbf{k} \in \text{Im}(\mathbb{H}),$$

define

$$u(q) = u(b\mathbf{i} + c\mathbf{j} + d\mathbf{k}) := \min\{\tilde{B}(b), \tilde{C}(c), \tilde{D}(d)\}.$$

Then u determines a fuzzy subset of $\text{Im}(\mathbb{H})$, called a *fuzzy pure quaternion set*.

Definition 2.5 (Quaternion-valued fuzzy set (alternative viewpoint)). Let X be a nonempty universe. Define the quaternion unit ball

$$\mathbb{B}_{\mathbb{H}} := \{ q \in \mathbb{H} \mid \|q\| \leq 1 \}.$$

A quaternion-valued fuzzy set on X is a map

$$\beta : X \rightarrow \mathbb{B}_{\mathbb{H}}.$$

A standard scalarization is via the norm: for $\alpha \in [0, 1]$, the α -cut can be defined as

$$A_{\alpha} := \{x \in X \mid \|\beta(x)\| \geq \alpha\}.$$

(Additional structure—e.g. a chosen order on $\mathbb{H}$ or operations on $\mathbb{B}_{\mathbb{H}}$ —is specified depending on the intended application.)

Definition 2.6 (Quaternion neutrosophic number (QNN)). A *single-valued quaternion neutrosophic number* is a pure quaternion of the form

$$q = T\mathbf{i} + I\mathbf{j} + F\mathbf{k} \in \mathbb{H}_0,$$

where the coefficients

$$T, I, F \in [0, 1]$$

correspond, respectively, to the truth-membership degree, the indeterminacy-membership degree, and the falsity-membership degree. These components are assumed to satisfy

$$0 \leq T + I + F \leq 3.$$

Thus, the three neutrosophic degrees are encoded into the $\mathbf{i}$ -, $\mathbf{j}$ -, and $\mathbf{k}$ -components of a pure quaternion.

Definition 2.7 (Quaternion neutrosophic set (QNS)). Let U be a nonempty universe. A *single-valued quaternion neutrosophic set* on U is determined by three mappings

$$TR_A, IND_A, FA_A : U \longrightarrow [0, 1],$$

where $TR_A(x)$, $IND_A(x)$, and $FA_A(x)$ denote the truth-membership, indeterminacy-membership, and falsity-membership degrees of $x \in U$, respectively. These functions are required to satisfy

$$0 \leq TR_A(x) + IND_A(x) + FA_A(x) \leq 3, \quad \forall x \in U.$$

Equivalently, the same information can be represented by a pure-quaternion-valued mapping

$$q_A : U \longrightarrow \mathbb{H}_0$$

defined by

$$q_A(x) := TR_A(x)\mathbf{i} + IND_A(x)\mathbf{j} + FA_A(x)\mathbf{k}.$$

Hence, a quaternion neutrosophic set may also be written as

$$A = \{(x, TR_A(x)\mathbf{i}, IND_A(x)\mathbf{j}, FA_A(x)\mathbf{k}) \mid x \in U\}.$$

This formulation stores the three neutrosophic components of each element as the imaginary components of a single pure quaternion.

3. Main Results of the paper

This section establishes the principal theoretical results of the paper.

3.1 Quaternion Fuzzy Graph

A quaternion fuzzy graph assigns unit-ball quaternions to vertices and edges, with edge magnitude bounded by the corresponding endpoint magnitudes. The motivation for quaternion fuzzy graphs is that ordinary fuzzy graphs use only scalar membership values. Quaternion labels allow vertices and edges to carry multi-component information, while the norm condition preserves the usual fuzzy-graph compatibility.

Definition 3.1 (Quaternion fuzzy graph). Let $G^* = (V, E)$ be a crisp graph (undirected or directed). A *quaternion fuzzy graph* (QFG) on G^* is a pair

$$G = (\rho, \beta),$$

where

$$\rho : V \rightarrow \mathbb{B}_{\mathbb{H}}, \quad \beta : E \rightarrow \mathbb{B}_{\mathbb{H}},$$

such that for every edge $uv \in E$ one has the compatibility constraint

$$\|\beta(uv)\| \leq \min\{\|\rho(u)\|, \|\rho(v)\|\}.$$

In the directed case, $uv \in E$ means an ordered pair (u, v) and the condition is imposed for each directed arc.

Example 3.2 (A concrete quaternion fuzzy graph). Let the underlying crisp undirected graph be the path

$$G^* = (V, E), \quad V = \{v_1, v_2, v_3\}, \quad E = \{v_1v_2, v_2v_3\}.$$

Define the vertex labeling $\rho : V \rightarrow \mathbb{B}_{\mathbb{H}}$ and the edge labeling $\beta : E \rightarrow \mathbb{B}_{\mathbb{H}}$ by

$$\rho(v_1) = 0.9, \quad \rho(v_2) = 0.6 + 0.3\mathbf{i}, \quad \rho(v_3) = 0.5 + 0.4\mathbf{j},$$

and

$$\beta(v_1v_2) = 0.4 + 0.1\mathbf{i}, \quad \beta(v_2v_3) = 0.3 + 0.2\mathbf{k}.$$

Since all other vertex pairs are non-edges, β is specified only on E . The corresponding quaternion fuzzy graph is illustrated in Figure 1.

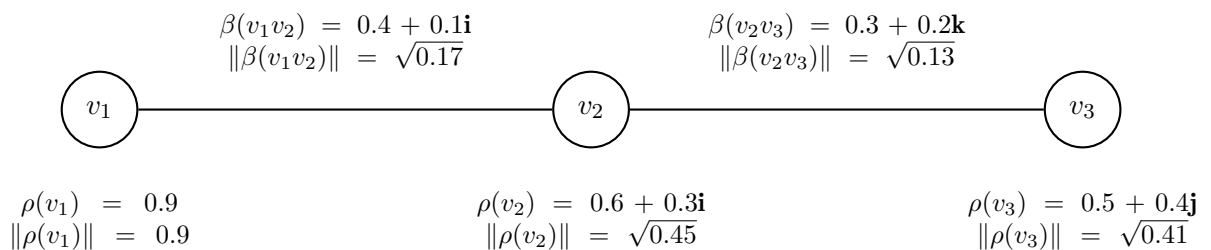


Fig. 1. A concrete quaternion fuzzy graph on the path $V = \{v_1, v_2, v_3\}$ and $E = \{v_1v_2, v_2v_3\}$.

First, all assigned values belong to the quaternion unit ball

$$\mathbb{B}_{\mathbb{H}} = \{q \in \mathbb{H} \mid \|q\| \leq 1\}.$$

Indeed,

$$\|\rho(v_1)\| = 0.9, \quad \|\rho(v_2)\| = \sqrt{0.6^2 + 0.3^2} = \sqrt{0.45} \leq 1,$$

and

$$\|\rho(v_3)\| = \sqrt{0.5^2 + 0.4^2} = \sqrt{0.41} \leq 1.$$

Similarly,

$$\|\beta(v_1v_2)\| = \sqrt{0.4^2 + 0.1^2} = \sqrt{0.17} \leq 1,$$

and

$$\|\beta(v_2v_3)\| = \sqrt{0.3^2 + 0.2^2} = \sqrt{0.13} \leq 1.$$

Next, the compatibility condition in Definition 3.1 is satisfied. For the edge v_1v_2 , we have

$$\|\beta(v_1v_2)\| = \sqrt{0.17} \leq \min\{\|\rho(v_1)\|, \|\rho(v_2)\|\} = \min\{0.9, \sqrt{0.45}\} = \sqrt{0.45}.$$

For the edge v_2v_3 , we obtain

$$\|\beta(v_2v_3)\| = \sqrt{0.13} \leq \min\{\|\rho(v_2)\|, \|\rho(v_3)\|\} = \min\{\sqrt{0.45}, \sqrt{0.41}\} = \sqrt{0.41}.$$

Therefore, $G = (\rho, \beta)$ is a quaternion fuzzy graph on G^* .

Theorem 3.3 (QFS structure and scalar reduction). *Let $G = (\rho, \beta)$ be a quaternion fuzzy graph on $G^* = (V, E)$.*

(i) (V, ρ) and (E, β) are quaternion-valued fuzzy sets.

(ii) Define real-valued functions

$$\rho_{\mathbb{R}}(u) := \|\rho(u)\| \quad (u \in V), \quad \beta_{\mathbb{R}}(uv) := \|\beta(uv)\| \quad (uv \in E).$$

Then $G_{\mathbb{R}} := (\rho_{\mathbb{R}}, \beta_{\mathbb{R}})$ is a (Rosenfeld) fuzzy graph on G^* , i.e.

$$\beta_{\mathbb{R}}(uv) \leq \min\{\rho_{\mathbb{R}}(u), \rho_{\mathbb{R}}(v)\} \quad (\forall uv \in E).$$

Proof. (i) is immediate from the codomains $\rho(V) \subseteq \mathbb{B}_{\mathbb{H}}$ and $\beta(E) \subseteq \mathbb{B}_{\mathbb{H}}$.

(ii) Since $\mathbb{B}_{\mathbb{H}} = \{q : \|q\| \leq 1\}$, we have $\rho_{\mathbb{R}}(u) \in [0, 1]$ and $\beta_{\mathbb{R}}(uv) \in [0, 1]$. The QFG constraint in Definition 3.1 gives

$$\beta_{\mathbb{R}}(uv) = \|\beta(uv)\| \leq \min\{\|\rho(u)\|, \|\rho(v)\|\} = \min\{\rho_{\mathbb{R}}(u), \rho_{\mathbb{R}}(v)\},$$

which is exactly the fuzzy-graph inequality. Hence $G_{\mathbb{R}}$ is a fuzzy graph. $\square$

Theorem 3.4 (Fuzzy graphs embed into quaternion fuzzy graphs). *Let $G^* = (V, E)$ be a crisp graph and let $G_{\mathbb{R}} = (\rho_{\mathbb{R}}, \beta_{\mathbb{R}})$ be a (Rosenfeld) fuzzy graph on G^* . Define the canonical embedding $\iota : [0, 1] \hookrightarrow \mathbb{H}$ by $\iota(t) := t$ (a real quaternion), and set*

$$\rho := \iota \circ \rho_{\mathbb{R}} : V \rightarrow \mathbb{B}_{\mathbb{H}}, \quad \beta := \iota \circ \beta_{\mathbb{R}} : E \rightarrow \mathbb{B}_{\mathbb{H}}.$$

Then $G = (\rho, \beta)$ is a quaternion fuzzy graph on G^* , and the scalar reduction of Theorem 3.3 satisfies

$$\|\rho(u)\| = \rho_{\mathbb{R}}(u), \quad \|\beta(uv)\| = \beta_{\mathbb{R}}(uv).$$

Consequently, quaternion fuzzy graphs strictly generalize fuzzy graphs.

Proof. Because $\rho_{\mathbb{R}}(u), \beta_{\mathbb{R}}(uv) \in [0, 1]$, we have $\|\rho(u)\| = \|\rho_{\mathbb{R}}(u)\| = \rho_{\mathbb{R}}(u) \leq 1$ and similarly $\|\beta(uv)\| \leq 1$, so ρ and β map into $\mathbb{B}_{\mathbb{H}}$. Moreover, for each $uv \in E$,

$$\|\beta(uv)\| = \beta_{\mathbb{R}}(uv) \leq \min\{\rho_{\mathbb{R}}(u), \rho_{\mathbb{R}}(v)\} = \min\{\|\rho(u)\|, \|\rho(v)\|\},$$

so Definition 3.1 holds. The identities follow from $\|t\| = |t| = t$ for $t \in [0, 1] \subset \mathbb{R}$. $\square$

3.2 Quaternion Neutrosophic Graph

A quaternion neutrosophic graph labels vertices and edges by truth, indeterminacy, falsity degrees encoded as pure quaternions, generalizing neutrosophic graphs. Quaternion neutrosophic graphs are useful because truth, indeterminacy, and falsity can be encoded as one pure-quaternion label. Thus, the neutrosophic meaning is retained while the uncertainty information is represented in a unified algebraic form.

Definition 3.5 (Quaternion neutrosophic graph). Let $G^* = (V, E)$ be a crisp graph (undirected or directed). A (single-valued) quaternion neutrosophic graph (QNG) on G^* is a pair

$$G = (A, B),$$

where $A : V \rightarrow \mathbb{H}_0$ and $B : E \rightarrow \mathbb{H}_0$ are pure-quaternion labelings of the form

$$A(v) = T_V(v)\mathbf{i} + I_V(v)\mathbf{j} + F_V(v)\mathbf{k} \quad (v \in V),$$

$$B(uv) = T_E(uv)\mathbf{i} + I_E(uv)\mathbf{j} + F_E(uv)\mathbf{k} \quad (uv \in E),$$

with $T_V, I_V, F_V : V \rightarrow [0, 1]$ and $T_E, I_E, F_E : E \rightarrow [0, 1]$ satisfying

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3 \quad (\forall v \in V),$$

$$0 \leq T_E(uv) + I_E(uv) + F_E(uv) \leq 3 \quad (\forall uv \in E),$$

and the incidence constraints (for every $uv \in E$)

$$T_E(uv) \leq \min\{T_V(u), T_V(v)\}, \quad I_E(uv) \leq \min\{I_V(u), I_V(v)\}, \quad F_E(uv) \geq \max\{F_V(u), F_V(v)\}.$$

In the directed case, $uv \in E$ denotes an arc (u, v) and the constraints are imposed on each arc.

Example 3.6 (A concrete quaternion neutrosophic graph). Let the underlying crisp (undirected) graph be the path

$$G^* = (V, E), \quad V = \{v_1, v_2, v_3\}, \quad E = \{v_1v_2, v_2v_3\}.$$

Define the vertex labeling $A : V \rightarrow \mathbb{H}_0$ by

$$A(v_1) = 0.8\mathbf{i} + 0.3\mathbf{j} + 0.2\mathbf{k},$$

$$A(v_2) = 0.6\mathbf{i} + 0.5\mathbf{j} + 0.4\mathbf{k},$$

$$A(v_3) = 0.7\mathbf{i} + 0.2\mathbf{j} + 0.6\mathbf{k},$$

so that (T_V, I_V, F_V) on V is given by

$$(T_V(v_1), I_V(v_1), F_V(v_1)) = (0.8, 0.3, 0.2),$$

$$(T_V(v_2), I_V(v_2), F_V(v_2)) = (0.6, 0.5, 0.4),$$

$$(T_V(v_3), I_V(v_3), F_V(v_3)) = (0.7, 0.2, 0.6).$$

Clearly, for each $v \in V$ we have $0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3$.

Next define the edge labeling $B : E \rightarrow \mathbb{H}_0$ by

$$B(v_1v_2) = 0.5\mathbf{i} + 0.2\mathbf{j} + 0.5\mathbf{k}, \quad B(v_2v_3) = 0.4\mathbf{i} + 0.1\mathbf{j} + 0.7\mathbf{k}.$$

Thus,

$$(TR_E(v_1v_2), IND_E(v_1v_2), FA_E(v_1v_2)) = (0.5, 0.2, 0.5),$$

$$(TR_E(v_2v_3), IND_E(v_2v_3), FA_E(v_2v_3)) = (0.4, 0.1, 0.7),$$

and again $0 \leq TR_E(e) + IND_E(e) + FA_E(e) \leq 3$ for each $e \in E$.

Finally, we verify the incidence constraints in Definition 3.5. For $v_1v_2 \in E$,

$$TR_E(v_1v_2) = 0.5 \leq \min\{0.8, 0.6\} = 0.6, \quad IND_E(v_1v_2) = 0.2 \leq \min\{0.3, 0.5\} = 0.3,$$

$$FA_E(v_1v_2) = 0.5 \geq \max\{0.2, 0.4\} = 0.4.$$

For $v_2v_3 \in E$,

$$TR_E(v_2v_3) = 0.4 \leq \min\{0.6, 0.7\} = 0.6, \quad IND_E(v_2v_3) = 0.1 \leq \min\{0.5, 0.2\} = 0.2,$$

$$FA_E(v_2v_3) = 0.7 \geq \max\{0.4, 0.6\} = 0.6.$$

Hence $G = (A, B)$ is a quaternion neutrosophic graph on G^* .

A quaternion neutrosophic graph is presented in Figure 2.

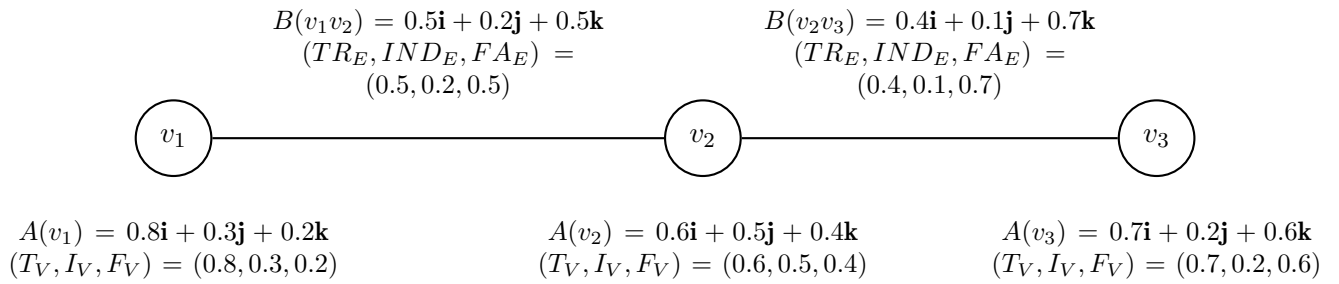


Fig. 2. A concrete quaternion neutrosophic graph on the path $V = \{v_1, v_2, v_3\}$ and $E = \{v_1v_2, v_2v_3\}$.

Theorem 3.7 (QNS structure and reduction to neutrosophic graphs). *Let $G = (A, B)$ be a quaternion neutrosophic graph on $G^* = (V, E)$.*

- (i) *The vertex labeling A determines a quaternion neutrosophic set on V and the edge labeling B determines a quaternion neutrosophic set on E .*
- (ii) *By extracting coefficients, i.e. writing*

$$A(v) = T_V(v)\mathbf{i} + I_V(v)\mathbf{j} + F_V(v)\mathbf{k}, \quad B(uv) = T_E(uv)\mathbf{i} + I_E(uv)\mathbf{j} + F_E(uv)\mathbf{k},$$

the pair of triples (T_V, I_V, F_V) and (T_E, I_E, F_E) defines a single-valued neutrosophic graph on G^ (in the sense of Definition 2.2).*

Proof. (i) is immediate: A (resp. B) is exactly the pure-quaternion encoding of three maps T_V, I_V, F_V (resp. T_E, I_E, F_E) satisfying the QNS bounds.

(ii) The coefficient maps take values in $[0, 1]$ and satisfy $0 \leq T + I + F \leq 3$ by Definition 3.5. The edge constraints in Definition 3.5 are exactly the componentwise neutrosophic incidence constraints used in Definition 2.2. Hence the extracted data form an SVNG on G^* . $\square$

Theorem 3.8 (Neutrosophic graphs embed into quaternion neutrosophic graphs). *Let $G^* = (V, E)$ be a crisp graph and let $G_N = (A_N, B_N)$ be a single-valued neutrosophic graph on G^* with membership triples*

$$(T_V, I_V, F_V) \text{ on } V, \quad (T_E, I_E, F_E) \text{ on } E.$$

Define $A : V \rightarrow \mathbb{H}_0$ and $B : E \rightarrow \mathbb{H}_0$ by quaternion encoding

$$A(v) := T_V(v)\mathbf{i} + I_V(v)\mathbf{j} + F_V(v)\mathbf{k}, \quad B(uv) := T_E(uv)\mathbf{i} + I_E(uv)\mathbf{j} + F_E(uv)\mathbf{k}.$$

Then $G = (A, B)$ is a quaternion neutrosophic graph on G^* . Moreover, the reduction of Theorem 3.7 recovers the original SVNG exactly. Therefore, quaternion neutrosophic graphs strictly generalize neutrosophic graphs.

Proof. All values lie in $[0, 1]$ and satisfy $0 \leq T + I + F \leq 3$ by the SVNG axioms. The componentwise inequalities for edges in Definition 3.5 hold because they are identical to the SVNG incidence constraints. Hence (A, B) is a QNG. Finally, coefficient extraction clearly returns the original functions $T_V, I_V, F_V, T_E, I_E, F_E$. $\square$

4. Illustrative Examples of Quaternion Neutrosophic Graphs in IT Management

This section presents two illustrative applications of quaternion neutrosophic graphs to information technology management, namely IT Service Management and IT Project Management. These examples show how service units, project components, and their operational or developmental dependencies can be modeled under uncertain and risk-sensitive conditions. In particular, the three components of a pure-quaternion label are interpreted as reliability, uncertainty, and risk information attached to IT-related entities and their dependency relations.

Application 4.1 (Application to IT Service Management). Consider a simplified IT Service Management (ITSM) (cf.[32]) setting consisting of three service-related units. Let

$$V = \{s_1, s_2, s_3\},$$

where s_1 denotes a service desk unit, s_2 denotes an incident management unit, and s_3 denotes a change management unit. The operational dependencies among these units are represented by

$$E = \{s_1s_2, s_2s_3\}.$$

Thus, the underlying crisp graph is the path $s_1 - s_2 - s_3$.

In this example, the truth-membership component T is interpreted as the degree of confirmed reliability or operational readiness. The indeterminacy-membership component I expresses incomplete or unclear information about the current service status. The falsity-membership component F represents operational risk, failure tendency, or unavailability. Hence a pure-quaternion label of the form

$$T\mathbf{i} + I\mathbf{j} + F\mathbf{k}$$

compactly records reliability, uncertainty, and risk in a single algebraic object.

Define the vertex labeling $A : V \rightarrow \mathbb{H}_0$ by

$$\begin{aligned} A(s_1) &= 0.85\mathbf{i} + 0.25\mathbf{j} + 0.15\mathbf{k}, \\ A(s_2) &= 0.80\mathbf{i} + 0.30\mathbf{j} + 0.20\mathbf{k}, \\ A(s_3) &= 0.70\mathbf{i} + 0.35\mathbf{j} + 0.30\mathbf{k}. \end{aligned}$$

Equivalently, the corresponding vertex triples are

$$\begin{aligned} (T_V(s_1), I_V(s_1), F_V(s_1)) &= (0.85, 0.25, 0.15), \\ (T_V(s_2), I_V(s_2), F_V(s_2)) &= (0.80, 0.30, 0.20), \\ (T_V(s_3), I_V(s_3), F_V(s_3)) &= (0.70, 0.35, 0.30). \end{aligned}$$

For each $s \in V$, these values satisfy

$$0 \leq T_V(s) + I_V(s) + F_V(s) \leq 3.$$

Next, define the edge labeling $B : E \rightarrow \mathbb{H}_0$ by

$$B(s_1s_2) = 0.75\mathbf{i} + 0.20\mathbf{j} + 0.25\mathbf{k}, \quad B(s_2s_3) = 0.65\mathbf{i} + 0.25\mathbf{j} + 0.35\mathbf{k}.$$

Thus,

$$(TR_E(s_1s_2), IND_E(s_1s_2), FA_E(s_1s_2)) = (0.75, 0.20, 0.25),$$

and

$$(TR_E(s_2s_3), IND_E(s_2s_3), FA_E(s_2s_3)) = (0.65, 0.25, 0.35).$$

Again, for every $e \in E$,

$$0 \leq TR_E(e) + IND_E(e) + FA_E(e) \leq 3.$$

It remains to check the quaternion neutrosophic incidence conditions. For the edge s_1s_2 , we have

$$TR_E(s_1s_2) = 0.75 \leq \min\{T_V(s_1), T_V(s_2)\} = \min\{0.85, 0.80\} = 0.80,$$

$$IND_E(s_1s_2) = 0.20 \leq \min\{I_V(s_1), I_V(s_2)\} = \min\{0.25, 0.30\} = 0.25,$$

and

$$FA_E(s_1s_2) = 0.25 \geq \max\{F_V(s_1), F_V(s_2)\} = \max\{0.15, 0.20\} = 0.20.$$

For the edge s_2s_3 , we similarly obtain

$$TR_E(s_2s_3) = 0.65 \leq \min\{T_V(s_2), T_V(s_3)\} = \min\{0.80, 0.70\} = 0.70,$$

$$IND_E(s_2s_3) = 0.25 \leq \min\{I_V(s_2), I_V(s_3)\} = \min\{0.30, 0.35\} = 0.30,$$

and

$$FA_E(s_2s_3) = 0.35 \geq \max\{F_V(s_2), F_V(s_3)\} = \max\{0.20, 0.30\} = 0.30.$$

Therefore, $G = (A, B)$ is a quaternion neutrosophic graph.

This construction illustrates how ITSM units and their operational dependencies can be evaluated by more than a single scalar reliability score. Each unit and each dependency link carries three coordinated pieces of information: confirmed operational strength, ambiguity in the available status information, and failure-related risk. For example, the edge s_2s_3 may describe the dependency between incident management and change management, where incident resolution may be affected by change approval, rollback capability, release coordination, and incomplete information about ongoing modifications.

Application 4.2 (Application to IT Project Management). Consider a simplified IT project management network (cf.[33]). Let

$$V = \{p_1, p_2, p_3\},$$

where p_1 represents requirements analysis, p_2 represents software development, and p_3 represents testing and quality assurance. The dependency relations among these project components are given by

$$E = \{p_1p_2, p_2p_3\}.$$

Thus, the crisp structure is the path

$$p_1 - p_2 - p_3.$$

In this interpretation, the truth-membership degree T measures the extent to which a project component or dependency is reliable, feasible, or sufficiently confirmed. The indeterminacy-membership degree I represents ambiguity arising from incomplete requirements, uncertain estimates, unresolved dependencies, or unclear technical assumptions. The falsity-membership degree F expresses risk, infeasibility, delay possibility, or potential failure. Consequently, the label

$$Ti + Ij + Fk$$

serves as a compact quaternionic representation of the uncertainty profile of a project element or dependency.

Define $A : V \rightarrow \mathbb{H}_0$ by

$$A(p_1) = 0.78\mathbf{i} + 0.40\mathbf{j} + 0.20\mathbf{k},$$

$$A(p_2) = 0.72\mathbf{i} + 0.35\mathbf{j} + 0.25\mathbf{k},$$

$$A(p_3) = 0.68\mathbf{i} + 0.30\mathbf{j} + 0.35\mathbf{k}.$$

Equivalently,

$$(T_V(p_1), I_V(p_1), F_V(p_1)) = (0.78, 0.40, 0.20),$$

$$(T_V(p_2), I_V(p_2), F_V(p_2)) = (0.72, 0.35, 0.25),$$

$$(T_V(p_3), I_V(p_3), F_V(p_3)) = (0.68, 0.30, 0.35).$$

For every $p \in V$, these vertex values satisfy

$$0 \leq T_V(p) + I_V(p) + F_V(p) \leq 3.$$

Define the edge labeling $B : E \rightarrow \mathbb{H}_0$ by

$$B(p_1p_2) = 0.65\mathbf{i} + 0.30\mathbf{j} + 0.30\mathbf{k}, \quad B(p_2p_3) = 0.60\mathbf{i} + 0.25\mathbf{j} + 0.40\mathbf{k}.$$

Hence,

$$(TR_E(p_1p_2), IND_E(p_1p_2), FA_E(p_1p_2)) = (0.65, 0.30, 0.30),$$

and

$$(TR_E(p_2p_3), IND_E(p_2p_3), FA_E(p_2p_3)) = (0.60, 0.25, 0.40).$$

These edge values also satisfy

$$0 \leq TR_E(e) + IND_E(e) + FA_E(e) \leq 3, \quad \forall e \in E.$$

We now verify the incidence conditions. For the dependency p_1p_2 , which connects requirements analysis to software development, one has

$$TR_E(p_1p_2) = 0.65 \leq \min\{T_V(p_1), T_V(p_2)\} = \min\{0.78, 0.72\} = 0.72,$$

$$IND_E(p_1p_2) = 0.30 \leq \min\{I_V(p_1), I_V(p_2)\} = \min\{0.40, 0.35\} = 0.35,$$

and

$$FA_E(p_1p_2) = 0.30 \geq \max\{F_V(p_1), F_V(p_2)\} = \max\{0.20, 0.25\} = 0.25.$$

For the dependency p_2p_3 , which connects software development to testing and quality assurance, one has

$$TR_E(p_2p_3) = 0.60 \leq \min\{T_V(p_2), T_V(p_3)\} = \min\{0.72, 0.68\} = 0.68,$$

$$IND_E(p_2p_3) = 0.25 \leq \min\{I_V(p_2), I_V(p_3)\} = \min\{0.35, 0.30\} = 0.30,$$

and

$$FA_E(p_2p_3) = 0.40 \geq \max\{F_V(p_2), F_V(p_3)\} = \max\{0.25, 0.35\} = 0.35.$$

Therefore, $G = (A, B)$ is a quaternion neutrosophic graph.

This example shows that quaternion neutrosophic graphs can represent project components and inter-component dependencies under uncertainty. For instance, the edge p_2p_3 may indicate that testing quality depends on development completeness, defect density, test-environment readiness, build stability, and unresolved technical issues. The relatively large falsity component of this edge reflects the possibility that unstable deliverables, delayed fixes, or remaining defects may negatively affect the testing phase.

5. Discussion

Quaternion fuzzy graphs and quaternion neutrosophic graphs provide a compact algebraic framework for representing multidimensional, directional, and inconsistent forms of uncertainty in network structures. In particular, quaternion fuzzy graphs extend ordinary fuzzy graphs by allowing vertex and edge labels to take values in the quaternion unit ball, while quaternion neutrosophic graphs encode truth, indeterminacy, and falsity degrees as pure-quaternion components. These features make the proposed models suitable for describing complex systems in which relationships are not only graded but also multidimensional and uncertainty-aware.

For reference, Table 2 compares fuzzy graphs with quaternion fuzzy graphs (QFGs), while Table 3 compares single-valued neutrosophic graphs with quaternion neutrosophic graphs (QNGs).

Table 2
Comparison of fuzzy graphs and quaternion fuzzy graphs (QFGs).

Aspect	Fuzzy graph	Quaternion fuzzy graph (QFG)
Label codomain	Scalar grades in $[0, 1]$.	Quaternion grades in the unit ball $\mathbb{B}_{\mathbb{H}} = \{q \in \mathbb{H} : \ q\ \leq 1\}$.
Vertex labeling	$\rho : V \rightarrow [0, 1]$.	$\rho : V \rightarrow \mathbb{B}_{\mathbb{H}}$.
Edge labeling	$\beta : E \rightarrow [0, 1]$, equivalently $\beta : V \times V \rightarrow [0, 1]$ with support on E .	$\beta : E \rightarrow \mathbb{B}_{\mathbb{H}}$.
Incidence constraint	$\beta(uv) \leq \min\{\rho(u), \rho(v)\}$ for all $uv \in E$.	$\ \beta(uv)\ \leq \min\{\ \rho(u)\ , \ \rho(v)\ \}$ for all $uv \in E$.
Relationship	Serves as the scalar baseline model.	Extends fuzzy graphs by embedding each $t \in [0, 1]$ as the real quaternion $t \in \mathbb{H}$. Conversely, the scalarization $\tilde{\rho} = \ \rho\ $, $\tilde{\beta} = \ \beta\ $ yields an ordinary fuzzy graph.

6. Conclusion

In this paper, we introduced two quaternion-based extensions of uncertainty-aware graph models. The first one is the *quaternion fuzzy graph*, in which vertices and edges are labeled by elements of the quaternion unit ball. Its incidence condition is formulated through the quaternion norm, so that the magnitude of each edge label is bounded by the magnitudes of its incident vertex labels. This construction preserves the basic compatibility principle of ordinary fuzzy graphs while allowing vertex and edge information to have a multi-component quaternionic form.

The second model is the *quaternion neutrosophic graph*, where the truth, indeterminacy, and falsity degrees of vertices and edges are encoded as pure-quaternion components. This representation

Table 3

Comparison of single-valued neutrosophic graphs and quaternion neutrosophic graphs (QNGs).

Aspect	Single-valued neutrosophic graph	Quaternion neutrosophic graph (QNG)
Vertex labels	Triples $(T_V(v), I_V(v), F_V(v)) \in [0, 1]^3$.	Pure quaternions $A(v) = T_V(v)\mathbf{i} + I_V(v)\mathbf{j} + F_V(v)\mathbf{k} \in \mathbb{H}_0$.
Edge labels	Triples $(TR_E(uv), IND_E(uv), FA_E(uv)) \in [0, 1]^3$.	Pure quaternions $B(uv) = TR_E(uv)\mathbf{i} + IND_E(uv)\mathbf{j} + FA_E(uv)\mathbf{k} \in \mathbb{H}_0$.
Incidence constraints	$TR_E(uv) \leq \min\{T_V(u), T_V(v)\},$ $IND_E(uv) \leq \min\{I_V(u), I_V(v)\},$ $FA_E(uv) \geq \max\{F_V(u), F_V(v)\}.$	The same componentwise constraints are encoded through the $\mathbf{i}$ -, $\mathbf{j}$ -, and $\mathbf{k}$ -components.
Relationship	Serves as the componentwise neutrosophic baseline model.	Extends neutrosophic graphs through the canonical identification $(T, I, F) \longleftrightarrow T\mathbf{i} + I\mathbf{j} + F\mathbf{k}.$ Conversely, projection onto the $\mathbf{i}$ -, $\mathbf{j}$ -, and $\mathbf{k}$ -components recovers the underlying neutrosophic graph.

gives an algebraic way to combine three types of uncertainty information in a single label of the form $T\mathbf{i} + I\mathbf{j} + F\mathbf{k}$. We showed that the proposed models naturally extend classical fuzzy graphs and single-valued neutrosophic graphs through canonical embeddings and componentwise reductions. We also presented illustrative examples in IT Service Management and IT Project Management, showing how reliability, ambiguity, and risk can be represented simultaneously for service units, project components, and their dependency relations.

Future work may proceed in several directions. First, quaternion fuzzy graphs and quaternion neutrosophic graphs can be extended by using plithogenic structures, where multiple attributes and contradiction degrees are explicitly incorporated. Second, the proposed models may be generalized to hypergraph-based frameworks in order to describe higher-order relations among more than two vertices. Third, hierarchical and nested dependency structures can be investigated through superhypergraph-based extensions. These directions may provide more expressive tools for modeling complex systems in which multidimensional uncertainty, dependency strength, and inconsistent information occurs simultaneously. Moreover, the concepts proposed in this paper can also be extended by using frameworks such as HyperSoft Sets and SuperHyperSoft Sets. Future research may also explore engineering applications in smart grids, robotics, cyber-physical systems, reliability engineering, telecommunications, software engineering, risk analysis, and intelligent transportation systems under multidimensional uncertainty. In addition, quantitative analysis based on computational experiments remains an important topic for future work.

Acknowledgement

This research was not funded by any grant.

Conflicts of Interest

The authors declare no conflicts of interest.

Declaration of Generative AI and AI-Assisted Technologies in the Manuscript Preparation Process

During the preparation of this work, the authors used AI-assisted tools ChatGPT-4 to enhance readability and improve language structure. These tools were not used to generate scientific content, analyses,

or conclusions. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the final manuscript. No generative AI tools were used to create or modify the study figures or artwork.

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